Diffraction from Two Parallel Slits

Abstract

The diffraction pattern from a pair of parallel optical slits illuminated with 650 nm diode laser light was characterized. The diffraction pattern showed characteristics attributable both to the width of the individual slits and to the separation between the two slits. The width of the slits was deduced to be $0.083 \pm 0.005\ mm$ and and their separation to be $0.253 \pm 0.002\ mm$, in good agreement with the nominal values.

Introduction

Diffraction of light through a pair of slits was first observed by Thomas Young in 1801. Two centuries later, the advent of lasers as cheap, widely available sources of coherent monochromatic light has made such observations easy and offer the chance for reasonable precision tests of diffraction theory. We have studied the diffraction patterns from both fixed and adjustable slits and compared the findings with diffraction theory.

Huygens' wave theory of light predicts the diffraction that should be observed when a pair of equal width slits, width w and center-center separation d, are illuminated with plane parallel light of wavelength λ. Diffraction means that the light is spread by its passage through the slits and emerges over a spread of exit angles producing a pattern of intensity as a function of angle as in Figure 1.

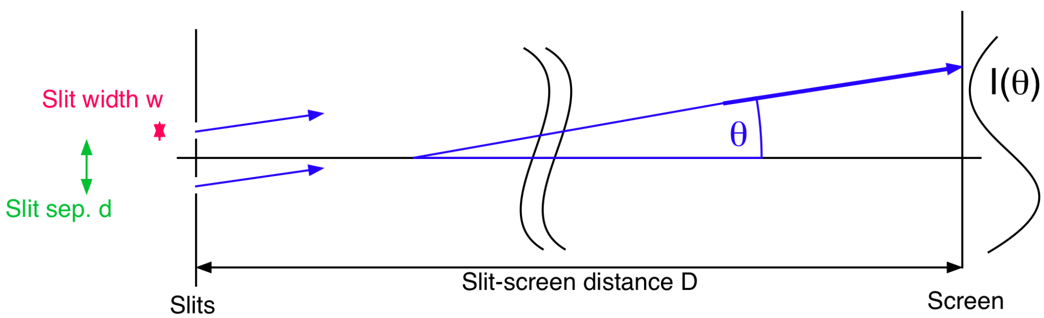

Fig. 1: Diffraction Geometry

As is shown in many physics texts[1] the diffraction pattern is expected to be the product of two different patterns. The large-scale structure comes from the individual slits and consists of a broad central maximum with maxima of decreasing amplitude spreading out on each side (Figure 2a). Within that, the pattern is a set of equal width, equal amplitude fringes whose separation is related to the slit separation d(Figure 2b). Multiplied together we get a set of equally spaced fringes whose inensities are controlled by the single-slit pattern (Figure 2c).

[1] E.g Townsend, *Quantum Physics: A Fundamental Approach to Modern Physics*, Chapter 1

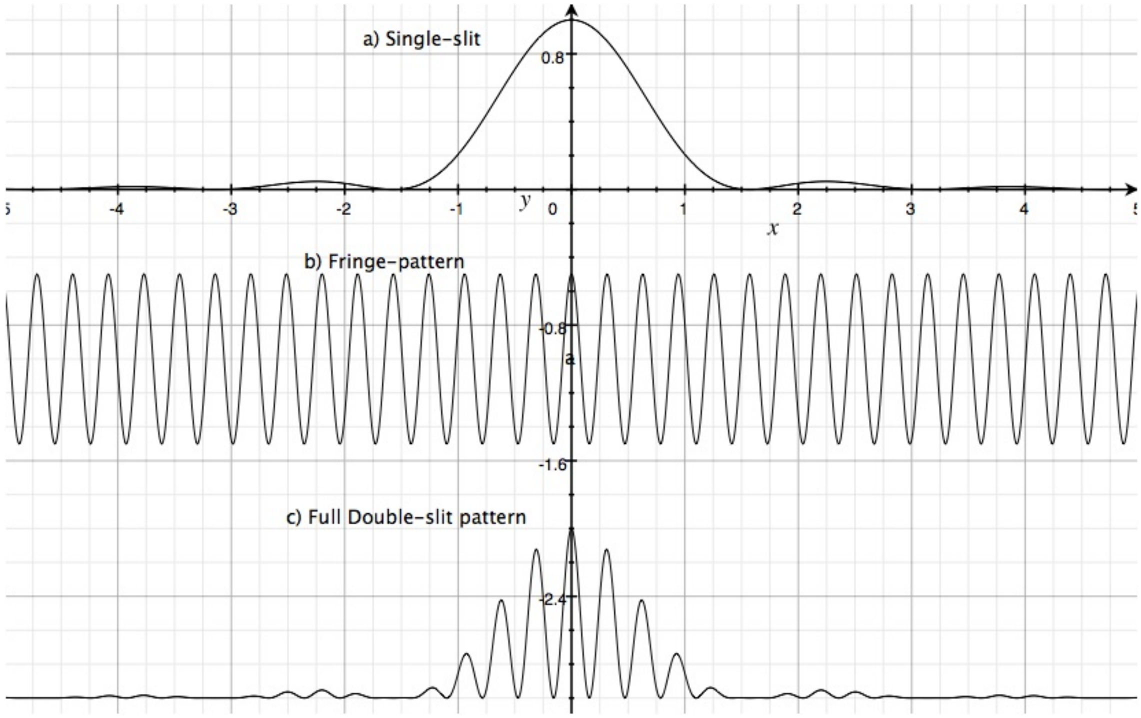

Fig. 2: Theoretical double-slit pattern

Methods

A Pasco OS-8525A 650 nm diode laser module and OS-8543 double-slit apparatus were mounted on a Pasco OS-8508 optical rail as shown in figure 1 below. The laser was aligned so that the beam was parallel to the optical rail and aligned with its axis. Alignment was verified by moving the slit assembly back and forward along the rail and ensuring that the beam always passed through the slit.

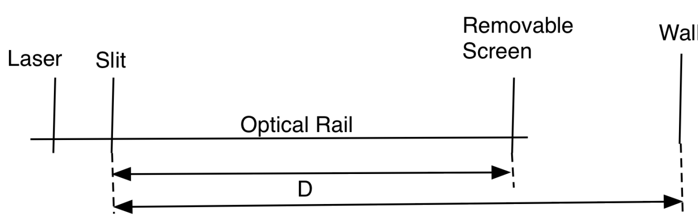

Fig. 3: Diffraction Apparatus

The slit assembly was adjusted so that light passed through either a pair of 0.08 mm wide slits with a separation of 0.25 mm (manufacturer's stated dimensions) or though the adjustable width slit towards the narrower end of its adjustment range. The resulting diffraction patterns were observed either on a screen mounted on the rail or on a white wall, perpendicular to the axis of the optical rail.

Diffraction was observed clearly from a separation (D in figure 1) of 200 cm up to the full 2m separation possible in our system. The large-scale single-slit pattern was clearly discernible at a separation of $D = 300 \pm 1$ mm but became more diffuse at very large separations, making measurements harder. The fine scale double-slit structure in the pattern, most clearly seen in the central single-slit maximum (region 0 in figure 2 below), became visible at about 700 mm and increased in clarity and separation up to the maximum separation, $D = 25000 \pm 1$ mm, possible with our apparatus. Patterns were recorded at both $D = 1000 \pm 1$ mm and at $D = 2500 \pm 1$ mm.

The diffraction pattern was recorded by taping ruler to the screen or wall and taking a photograph with an iPhone camera.

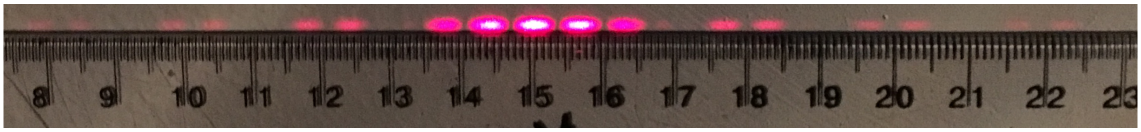

Fig. 4: Diffraction Pattern with scale. The scale is a plastic ruler calibrated in cm.

Results

The diffraction pattern, Figure 4 above, showed characteristics of both single-slit and double slit diffraction, supporting the idea that there were two slits whose widths and separations were significantly greater than the wavelength of the light and yet small enough to exhibit significant diffraction. The bright fringes of the double-slit pattern are strongly modulated by the single-slit pattern. The fact that the third, sixth, and ninth bright fringes, counting out from the centre fringe at 15 cm, are missing shows clearly the effect of the single-slit pattern.

Double-slit pattern

Figure 5 below shows the pattern relabelled to emphasize the pattern of equally space maxima and minima that is characteristic of a pair of slits. The sub-maxima were numbered from the central spot (spot 0 in figure 4), corresponding to the diffraction order n in the classic double-slit formula

$$d \sin \theta_n = n\lambda$$

where d is the slit separation, lambda the wavelength of light, and θ_n the angle between the straight-through light and the center of the nth maximum.

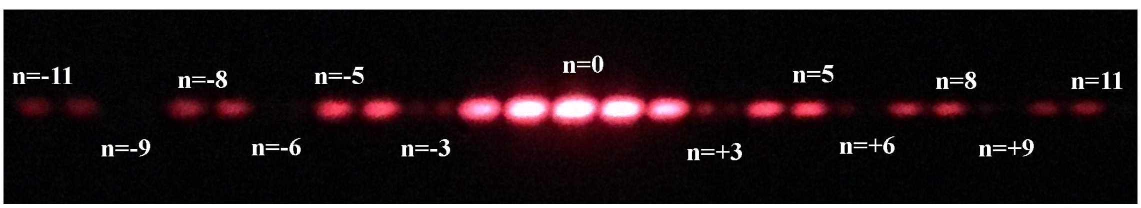

Fig. 5: Pattern annotated with indices of double-slit maxima.

In order to minimize the relative error, the distance between spot -11 and spot 11 was measured to be 144 ± 1 mm so that the separation between the center and the fifteenth maximum was $y_n = 72 \pm 0.5$ mm. The uncertainty is dominated by the difficulty of estimating the center of the spot rather than by the readability of the ruler.

Using the small angle approximation we find, neglecting the uncertainty in $D = 2500 \pm 1$ mm,

$$\theta_n \simeq \tan \theta_n = \frac{y_n}{D} = \frac{72 \pm 0.5\ mm}{2500\ mm} = 0.0288 \pm 0.0002\ radians.$$

This leads to a value for the slit separation

$$d = \frac{n\lambda}{\theta_n} = \frac{11 \times 650 \times 10^{-9}\ m}{0.0288 \pm 0.0002} = 0.248 \pm 0.002\ mm$$

which is consistent with just barely being able to see the two slits by eye and in good agreement with the nominal 0.25 mm value printed on the slit set.

Single-slit Pattern

Figure 6 below shows the active region from figure 3 annotated to emphasize the classic single-slit pattern of a wide, intense central maximum (maximum 0) bordered by narrower regions of successively decreasing intensity (maxima 1, 2, and 3). There are also clearly visible striations within the single-slit maxima, consistent with more than one slit.

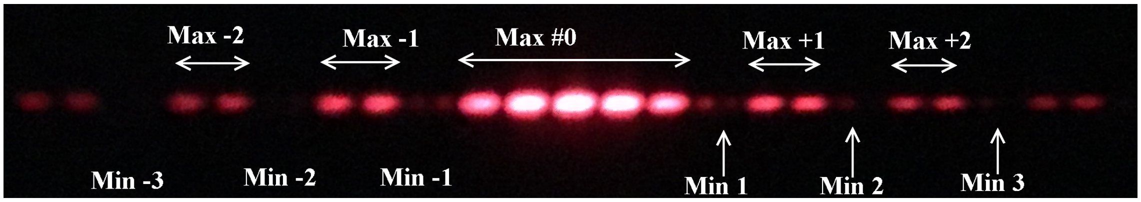

Fig. 6: Pattern annotated to show single-slit maxima and minima.

It is harder to localise the centers of the dark minima since they are quite wide. First we note that there is very good alignment of the single-slit minima with the third, sicth, and ninth maxima of the double-slit pattern. Since the maxima of the double slit pattern occur at angle θ_n given by

$$d \sin \theta_n = n\lambda$$

while the minima of the single-slit pattern are found at angles ϕ_m given by

$$w \sin \phi_m = m\lambda$$

we find that the m'th single-slit minimum coincides with the $3m$'th double slit maximum so that

$$\phi_m = \theta_{3m}$$

or

$$\frac{m}{w} = \frac{3m}{d},$$

which gives us $w = d/3$. This gives us a good approximation to the value of the width. In fact, since we can clearly tell that the $m = 3$ minimum coincides with the $n = 9$ maximum and is not displaced by as much as one half of a fringe width we can say that $3d = (9 \pm 0.5)\, w$ or

$$w = \frac{3d}{9 \pm 0.5} = \frac{3 \times 0.248\; mm}{9 \pm 0.5} = 0.083 \pm 0.005\; mm$$

This is probably as accurate as we can be with these data. Again, the computed value is in good agreement with the nominal 0.08 mm value printed on the slit set.

Adjustable Double Slit

The pattern from an adjustable double slit was also recorded for qualitative comparison. Like the fixed double slit it shows a marked single-slit pattern with a series of wide bright regions separated by narrow dark regions and the central region twice the width of the others. Within the single-slit envelope there are the equally spaced fringes of a double-slit pattern (Figure 5).

Fig. 7: Adjustable Double Slit

Discussion

The diffraction pattern exhibits clear characteristics of a finite-double slit. The fine structure of evenly spaced maxima and minima is modulated by a much coarser structure of maxima and minima in which the central maximum is twice as wide as the peripheral maxima.

It is interesting to note the details of what happens at a minimum of the single-slit pattern. Figure 8 below is an enlargement of the region including the central single-slit maximum and the first sub-maximum on the right. The minima have been marked with short vertical lines andthe intervening maxima numbered.

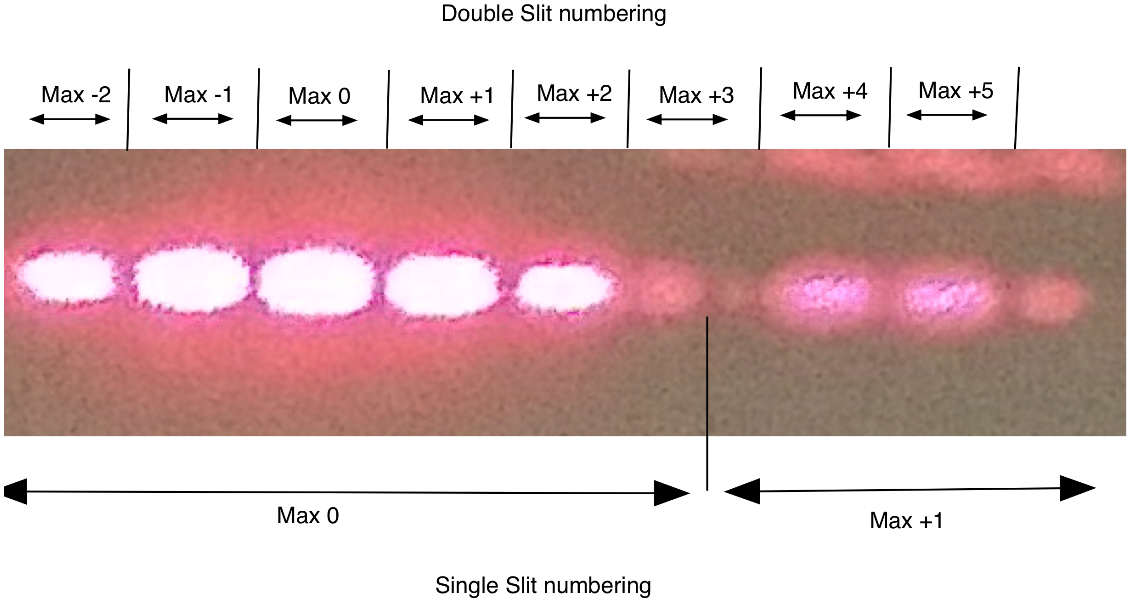

Fig. 8: Detail of center of pattern showing relation between single and double slit maxima.

The first minimum of the single-slit pattern lies on top of the third maximum of the double-slit pattern, introducing an extra minimum. We actually see that the single-slit minimum falls about 2/3 of the way through the third double-slit maximum, so that the single slit corresponds to a spacing that is slightly more than three times smaller than the double-slit spacing. This is, obviously, in agreement with our measurements.

It is important to take into account the ability of the single-slit pattern to add extra minima to the overall pattern. If we are not careful, it is easy to mis-count the number of double-slit maxima when a maximum is cut in half like this.

The pattern from the adjustable double slit (Figure 7) strongly resembles that of the fixed width double slit. The fact that the single-slit minima appear totally dark shows that the two slits are of the same width, so that the two waves that interfere destructively have the same amplitude. The new feature is the the double-slit fringes are no longer symmetric from top to bottom. It is most clearly seen in the central region that the fringes appear as slanted lines, closer together at the top of the picture than at the bottom. Since the separation between the fringes is inversely proprtional to the spacing between the two slits this tells us that the adjustable slits were wider apart at teh top than at the bottom. There is some left-right asymmetry to the pattern, with the right side appearing somewhat blurred compared to the left. This is especially apparent in the fringes inside the $n = \pm 1$ sub-maxima of the single-slit pattern. The one on the left continues to show clear fringe detail while there is no detail seen inside the one on the right. It was clear during the measurement that details at this level were quite variable depending on the precise aligment of laser and slit and it may indicate smudging on glass windows of the slit apparatus. Since it involves a change from the extreme left side of the picture to the extreme right it must correspond to some feature in the slit apparatus that is much smaller than the width of the individual slits.

Conclusion

We have shown the diffractino patterns from the fixed and adjustable double-slits in the Pasco apparatus are essentially perfectly described by the classical theory. Furthermore, we are able to extract information from the double-slit diffraction pattern with a precision that is limited by our ability to measure features in the diffraction pattern. The measured width and separation of 0.083 ± 0.005 mm and 0.253 ± 0.002 mm are in good agreement with the nominal values provided by the slit vendor.